2nd Day for Number Theory Ph.D Students

Centro Congressi Santa Elisabetta - Parma

April 12th, 2018

Anwar Mohammed

On Schinzel-Wójcik problem

The Schinzel–Wójcik problem consists in determing if given $a_1, \ldots, a_r \in \mathbb{Q}^* - \{\pm 1\}$, there exist infinitely many primes p such that they have the same multiplicative order modulo p. I will mention some results about Schinzel–Wójcik problem and Schinzel–Wójcik problem on average.

Cafferata Mattia

Orthogonality properties for a family of arithmetic functions

Let $\mu(n)$ be the Möbius function. The *Möbius randomness law* is a well-known but vague principle concerning the orthogonality of μ with any "reasonable" function $\xi(n)$, i.e. that

$$\sum_{n \leq N} \mu(n)\xi(n) = o\left(\sum_{n \leq N} |\xi_n|\right)$$

for every function $\xi(n)$ satisfying suitable hypotheses.
After recalling some classical and recent results in the context of the Sarnak Conjecture we discuss the possibility to extend them to a new family of arithmetic functions strictly related to $\mu(n)$.

Coscelli Edoardo

Stickelberger series and Iwasawa main conjecture for function fields

Let F be a global function field in characteristic $p > 0$. There exists many different types of L-functions that can be associated to F, such as the Artin L-functions, the Goss Zeta function or the $\mathfrak{p}$-adic L-functions. We will investigate the correlations between these analytic objects and the Stickelberger series, which is a formal power series whose coefficient lies in a suitable Galois algebra. In the second part of the talk we will study the Iwasawa extension generated by the $\mathfrak{p}$-torsion of a Hayes module and we will use the Stickelberger series to prove a *main conjecture* for the p-part of the class group.

Gatti Pietro

The Monodromy of a Semistable Family of Curves

For a semistable family of curves over the complex numbers, we describe explicitly the monodromy on the cohomology of the generic fiber. Using logarithmic geometry, we compute this cohomology group on the special fiber and then exploit its combinatorics to reconstruct the monodromy. As a result, we give a completely algebraic proof of the invariant cycles theorem for curves. Even though we deal with complex varieties and their topology, this study is fundamentally inspired by the work of Chiarellotto, Coleman, Di Proietto and Iovita on a p-adic analogue of this situation. The first part of the talk will be devoted to reviewing the theory of p-adic invariant cycles in order to justify our methodology.

Panozzo Simone

Characteristic series of U operator over the boundary of weight space

Starting from the work of Andreatta, Iovita and Pilloni, I will recall the modular interpretation of the reduction of characteristic series of the U_p operator acting on spaces of overconvergent modular forms. Starting from this viewpoint, I will construct a suitable basis, in some specific cases, which allows to write explicitly the action of the U_p operator, and to study some conjectural properties related to the Newton polygon of its characteristic series.

Sanna Carlo

Distribution of integral values for the ratio of two linear recurrences

Let $F(n)_{n\geq 1}$ and $G(n)_{n\geq 1}$ be two linear recurrences over a number field $\mathbb{K}$, and let $\mathfrak{R}$ be a finitely generated subring of $\mathbb{K}$. Furthermore, define the set

$$\mathcal{N} := \{n \geq 1 : G(n) \neq 0,\ F(n)/G(n) \in \mathfrak{R}\}.$$

Corvaja and Zannier proved that if F/G is not a linear recurrence, and if some other mild hypotheses are satisfied, then $\mathcal{N}$ has zero asymptotic density.

We improve this result by proving the upper bound

$$\#(\mathcal{N} \cap [1, x]) \ll x \cdot \left(\frac{\log\log x}{\log x}\right)^h,$$

for all $x \geq 3$, where h is a positive integer that can be computed in terms of F and G. The proof employs a quantitative version of Chebotarev density theorem, a lemma for a sieved set of integers, and bounds for the number of zeros of sparse polynomials in finite fields.

Also, we show that under the Hardy–Littlewood k-tuple conjecture our result is optimal except for the factor $\log\log x$.

Zaghloul Giamila

On the linear twist of degree-1 functions in the extended Selberg class

Let F be a degree-1 function in the extended Selberg class $\boldsymbol{S}^\sharp$ and $\alpha \in \mathbb{R}$. We study the main analytic properties of the linear twist $F(s, \alpha)$. Starting from the characterization of $\boldsymbol{S}_1^\sharp$ and the known properties of the Hurwitz-Lerch zeta function, we show that the linear twist satisfies a functional equation of Hurwitz-Lerch type. We also discuss some results on the polynomial growth and the distribution of the zeros.