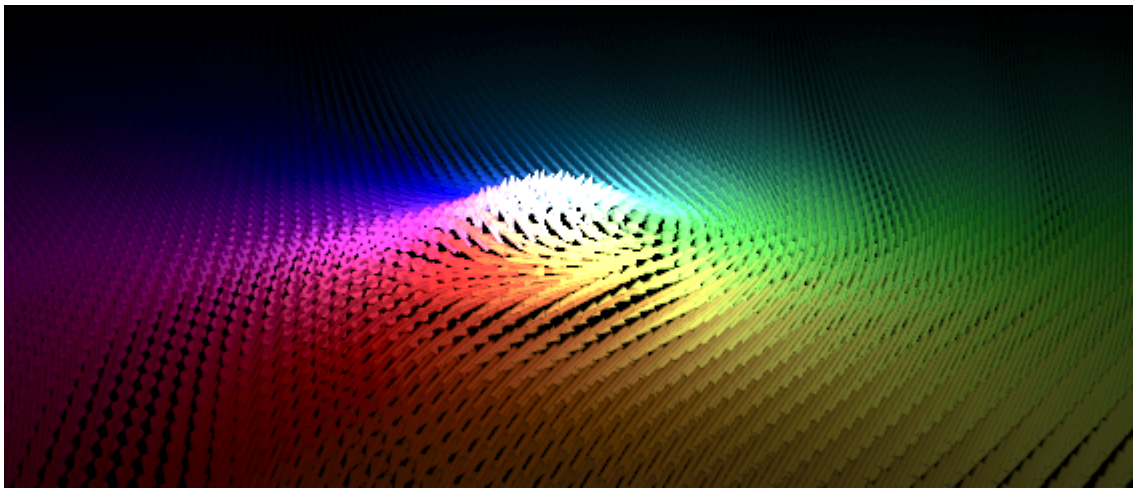

Modeling and Analysis in Nanomagnetism and beyond

(organized by M. Morini, C. Muratov, M. Novaga, G. Palatucci)

Book of abstracts

Aula G, Floor -1, Department of Mathematics and Computer Science, May 27–31, 2024

Contents

Skyrmions in ultrathin magnetic films: an overview

Monday 27, 9:30

Cyrill Muratov (University of Pisa)

Abstract. I will present an overview of the current results on existence and asymptotic properties of magnetic skyrmions defined as topologically nontrivial maps of degree +1 from the plane to a sphere which minimize a micromagnetic energy containing the exchange, perpendicular magnetic anisotropy and interfacial Dzyaloshinskii-Moriya interaction (DMI) terms. In ultrathin films, the stray field energy simply renormalizes the anisotropy constant at leading order, but in finite samples it also produces additional non-trivial contributions at the sample edges, promoting nontrivial spin textures. Starting with the whole space problem, I will first discuss the existence of single skyrmions as global energy minimizers at sufficiently small DMI strength. Then, using the quantitative rigidity of the harmonic maps I will present the asymptotic characterization of single skyrmion profiles both in infinite and finite samples. Lastly, I will touch upon the question of existence of multi-skyrmion solutions as minimizers with higher topological degree and present recent existence results obtained jointly with T. Simon and V. Slastikov.

Introduction to Mathematical Micromagnetics - I

Monday 27, 11:00 (via Teams)

Valeriy Slastikov (University of Bristol)

Abstract. In several lectures we cover some basic topics of mathematical micromagnetics. In Lecture 1 we define the micromagnetic energy and explain formation of some universal magnetic structures (domain walls, vortices, skyrmions) in the simplest setting. In Lecture two we study the influence of magnetostatic energy on magnetization patterns in ferromagnetic materials and derive several reduced models for thin ferromagnetic films. In Lecture 3 we want to understand formation of periodic structures in thin films with perpendicular anisotropy and appearance of microstructure/branching near the boundary of thick films.

The elastica functional as the critical Gamma-limit of the screened Gamow model

Monday 27, 14:00

Theresa Simon (University of Münster)

Abstract. I will consider the large mass limit of a nonlocal isoperimetric problem in two dimensions with screened Coulomb repulsion. In this regime, the competition between perimeter and the repulsion simplies to leading order by the nonlocal interaction localizing on the boundary of the sets. For an appropriate choice of screening constant, the surface area is exactly cancelled, requiring an analysis of the next order contribution. It turns out that then the nature of the prob-

lem changes from length minimization to curvature minimization: I will prove that the Gamma limit is given by (the relaxation of) the elastica functional, i.e., the integral over the squared curvature over the boundary.

Optimal size for magnetic domains in Thin Ferromagnetic Films

Monday 27, 15:00

Hans Knüpfer (University of Heidelberg)

Abstract. We consider a regime of large and ultra-thin ferromagnetic films with strong anisotropy and easy axis pointing out of the film plane. Starting from the full three-dimensional micromagnetic energy, using the framwork of the calculus of variations and asymptotic analysis, we identify (to leading order) the critical scaling where the phase transition from single domain states to multi-domain states such as bubble or maze patterns occurs. The results of the analysis also include a rigorous proof for a scaling for the domain size. We also derive a reduced model in the framework of Gamma-convergence. This is joint work with B. Brietzke, C. Muratov, F. Nolte and W. Shi.

Domain walls in ferromagnetic nanostrips

Monday 27, 16:30

Matteo Novaga (University of Pisa)

Abstract. I will discuss existence, uniqueness and symmetry of domain wall solutions arising as minimizers of an energy functional for infinitely long and thin ferromagnetic strips, when the magnetization is forced to lie in the film plane.

Topological spin textures stabilized by long-range dipolar interaction in ferromagnetic thin films and their applications

Tuesday 28, 9:30

Anne Bernand-Mantel (University of Toulouse)

Abstract. Topological spin textures are potential bit-encoding states for various information technology applications including high-density/high-speed memory and unconventional computing such as neuromorphic, probabilistic and reservoir computing. As a consequence, these systems have been widely studied experimentally and theoretically in recent years. Nevertheless, despite numerous attempts, a satisfactory theoretical description of these objects is still lacking today due to the highly non-trivial character of the magnetostatic interaction that plays a major role in determining the nature of magnetization patterns in ferromagnets. In this talk we will present a pedagogical overview of magnetization patterns existing in the case of ferromagnetic thin films. We will build a phase diagrams to locate magnetic patterns (uniform, stripe, helicoid, bubble, skyrmion...) as a function of the ferromagnetic layer thickness, applied magnetic field and other physical parameters. We will start with simplified "toy" models enabling physical insight of what a ferromagnet is and the origin of hysteresis (Stoner-Wohlfarth, thin wall models...). We will continue with results involving rigorous mathematical analysis. This talk will be illustrated with experimental and numerical observations. We will finally give a concrete example of a promising applications using topological spin textures for ultra-fast/ultra-low power physical random number generation with various applications from probabilistic AI to Monte Carlo based intensive calculations in finance. This is joint work with C. Muratov, T. Simon and V. Slastikov.

Introduction to Mathematical Micromagnetics - II

Tuesday 28, 11:00 via Teams

Valeriy Slastikov (University of Bristol)

Abstract. In several lectures we cover some basic topics of mathematical micromagnetics. In Lecture 1 we define the micromagnetic energy and explain formation of some universal magnetic structures (domain walls, vortices, skyrmions) in the simplest setting. In Lecture two we study the influence of magnetostatic energy on magnetization patterns in ferromagnetic materials and derive several reduced models for thin ferromagnetic films. In Lecture 3 we want to understand formation of periodic structures in thin films with perpendicular anisotropy and appearance of microstructure/branching near the boundary of thick films.

Magnetic dynamics in a ferrimagnetic alloy

Tuesday 28, 14:00

Joao Sampaio (University of Paris-Saclay)

Abstract.

Ferrimagnets, like antiferromagnets, are materials with two or more strongly-coupled spin populations, and display dynamics that are faster and qualitatively different from those of ferromagnets. This makes them interesting for applications, and has motivated a surge in interest for these materials in spintronics. The more complex dynamics presents, however, a challenge for the understanding the behaviour of ferrimagnetic magnetic textures. In this talk, I will present the work we have done on the experimental study and theoretical and numerical modelling of the dynamics of spin waves, domain walls, and skyrmions in Gd(Fe)Co, a ferrimagnetic alloy. This is joint work with E. Haltz, L. Berges, A. Mougin, S. Rohart and A. Thiaville.

Periodic striped states in Ising models with polynomial interactions

Tuesday 28, 15:00

Alessandro Giuliani (University of Roma Tre)

Abstract. I will discuss the problem of determining the ground states of $2D$ Ising models with nearest neighbor ferromagnetic and dipolar-like interactions, and I will review known results, including the proof of existence of periodic striped minimizers in the presence of anti-ferromagnetic polynomial interactions decaying like $1/(dist)^p$, with p larger than $4-\epsilon$. In the standard dipolar case, $p=3$, I will discuss the proof that the minimizers in the variational class of states whose domains walls are arbitrary collections of horizontal and/or vertical straight lines are periodic and striped. Based on joint works with Davide Fermi, Joel Lebowitz, Elliott Lieb, Robert Seiringer.

Minimizing micromagnetic maps with axial symmetry

Tuesday 28, 16:30

Giovanni Di Fratta (University Federico II Napoli)

Abstract. The talk focuses on the symmetry properties of global minimizers of a Dirichlet-type energy functional defined on the space of vector fields $H^1(S,T)$, where S and T are surfaces of revolution. The energy functional we present is closely related to a reduced model in the variational theory of micromagnetism for the analysis of magnetic skyrmions in curved thin films. We show that axially symmetric minimizers always exist. Furthermore, any coexisting minimizer must have line symmetry if the target surface T is never flat. As a result, the minimization problem is reduced to the computation of an optimal one-dimensional profile.

Field theory of a three-sublattice antiferromagnet

Wednesday 29, 9:30

Oleg Tchernyshyov (John Hopkins University)

Abstract. We present a field theory of a three-sublattice hexagonal antiferromagnet. The order parameter is the spin frame, an orthogonal triplet of vectors related to sublattice magnetizations and spin chirality. The exchange energy, quadratic in spin-frame gradients, has three coupling constants, only two of which manifest themselves in the bulk. As a result, the three spin-wave velocities satisfy a universal relation. Vortices generally have an elliptical shape with the eccentricity determined by the Lamé parameters.

Introduction to Mathematical Micromagnetics - III

Wednesday 29, 11:00 via Teams

Valeriy Slastikov (University of Bristol)

Abstract. In several lectures we cover some basic topics of mathematical micromagnetics. In Lecture 1 we define the micromagnetic energy and explain formation of some universal magnetic structures (domain walls, vortices, skyrmions) in the simplest setting. In Lecture two we study the influence of magnetostatic energy on magnetization patterns in ferromagnetic materials and derive several reduced models for thin ferromagnetic films. In Lecture 3 we want to understand formation of periodic structures in thin films with perpendicular anisotropy and appearance of microstructure/branching near the boundary of thick films.

Topological magnetic structures with homotopy groups of quadratic growth and exponential growth

Thursday 30, 9:30

Filipp Rybakov (Univ. Uppsala)

Abstract. In the first part of the talk, we will discuss new experimental and theoretical results on magnetic hopfions and skyrmions [1]. These topological structures form combinations and are classified by the homotopy group of quadratic growth [1], which we will discuss in more detail. In the second part, we will focus on the theory of non-abelian vortices in anisotropic magnets, where the classifying homotopy group is the free group [2].

1. F. Zheng, et al., Nature 623, 718 (2023)

2. F. N. Rybakov and O. Eriksson, arXiv:2205.15264 (2022)

Quantitative estimates for the Dirichlet energy of maps into the sphere

Thursday 30, 11:00

Melanie Rupflin (University of Oxford)

Abstract. The Dirichlet energy of maps from R^2 to S^2 is at least 4π times the degree, with equality if and only if the map is rational, i.e. given in coordinates by a meromorphic function. Hence it is natural to ask whether maps with small energy defect $\delta_v = E(v) - 4\pi \deg(v)$ are necessarily close to a rational map. In this talk we will explain how the fact that energy can concentrate on multiple scales means that this rigidity statement is actually false for maps of general degree. At the same time we present an approach that allows to measure and control the distance between such a map which shows multi-scale behaviour and the nearest generalised energy minimiser (which is a collection of rational maps that represent the behaviour at all relevant scales) and prove that this distance is controlled by a quantitative stability estimate of the form $\text{dist}^2 \leq C\delta_v(1 + |\log \delta_v|)$, which is indeed sharp.

Hopfions in magnetic crystals

Thursday 30, 14:00

Nikolai Kiselev (University of Juelich)

Abstract. In my talk, I will discuss three distinct types of three-dimensional topological magnetic solitons, which can be hosted by two different types of magnetic crystals:

- i) Hopfions in frustrated magnets [1].
- ii) Hopfion rings on skyrmion strings in chiral magnets [2].

- iii) Heliknoton in chiral magnets [3, 4, 5].

I will especially focus on the experimental observation of hopfion rings in B20-type FeGe samples, through high-resolution transmission electron microscopy. I will discuss various aspects of hopfion rings, including the protocol for hopfion ring nucleation, the diversity of configurations of hopfion rings linked with one or a few skyrmion strings, hopfion ring zero modes, etc.

1 Rybakov, F. N. et al. APL Mater. 10, 111113 (2022).

2 Zheng, F. et al., Nature 623, 718 (2023).

3 J-S. B. Tai, I.I. Smalyukh, Science 365, 1449–53 (2019).

4 R. Voinescu, J-S. B. Tai, I. I. Smalyukh, Phys. Rev. Lett. 125, 057201 (2020).

5 V.M. Kuchkin, et al., Front. Phys. 11, 1201018 (2023).

Existence and structure of 360-degree walls in thin films

Thursday 30, 15:00

Antonio Capella (Univ. Nacional Autónoma de México)

Abstract. Abstract: The 360-degree walls are topologically constrained one-dimensional minimizers of the micromagnetic energy in thin films. In this talk, I will present some results that demonstrate how the interplay between the non-local term in the energy and the topological constraint leads to the existence of 360-degree walls for all orientations, making a non-zero angle with the easy axis in the absence of an applied magnetic field. Geometrically, a 360-degree wall can be regarded as two separated 180-degree walls that interact through the stray field, resembling a sort of dipole-dipole interaction. Additionally, we will explore how this general structure depends on the parameter values. This talk is based on a joint work with Hans Knüpfer and Cyrill Muratov.

Chirality phases at the ferromagnet-helimagnet transition point

Thursday 30, 16:30

Marco Cicalese (Technical University of Munich)

Abstract. We introduce a classical ferromagnetic/antiferromagnetic lattice spin model. We focus on the excess energy of the system about the ground states in the vicinity of the ferromagnet/helimagnet transition point. Carrying out the Gamma-convergence analysis of this model as the lattice spacing vanishes we show the emergence of chirality domains and determine the optimal cost of a chirality transition. The discussion will primarily address the case of S^1-valued spins, and we will conclude with very recent progresses related to the S^2 case.

t.b.a.

Friday 31, 9:30

Lucia Scardia (Univ. Edimburgh)

Abstract. N/A

Normalized solutions and limit profiles of the Gross-Pitaevskii-Poisson equation

Friday 31, 11:00

Vitaly Moroz (University of Swansea)

Abstract. Gross-Pitaevskii-Poisson (GPP) equation is a nonlocal modification of the Gross-Pitaevskii equation with an attractive Coulomb-like term. It appears in the models of self-gravitating Bose-Einstein condensates proposed in cosmology and astrophysics to describe Cold Dark Matter and Boson Stars. We investigate the existence of prescribed mass (normalised) solutions to the GPP equation, paying special attention to the shape and asymptotic behaviour of the associated mass-energy relation curves and to the limit profiles of solutions at the endpoints of these curves. In particular, we show that after appropriate rescalings, the constructed normalized solutions converge either to a ground state of the Choquard equation, or to a compactly supported radial ground state of the integral Thomas-Fermi equation. In different regimes the constructed solutions include global minima, local but not global minima and unstable mountain-pass type solutions. This is a joint work with Riccardo Molle and Giuseppe Riey.