Moser's estimates for degenerate Kolmogorov equations with non-negative divergence lower order coefficients

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Abstract : Degenerate Kolmogorov equations arise in the theory of stochastic processes (e.g. the simplest non-trivial Kolmogorov operator is the infinitesimal generator of the Langevin's stochastic equation), kinetic theory (e.g. linear Fokker-Planck equations, non-linear Boltzmann-Landau equations) and mathematical finance (e.g. problem of pricing Asian options). The study of the regularity theory for weak solutions to this kind of equations is carried out paralleling the weak theory for parabolic equations, such as Sobolev and Caccioppoli inequalities, Moser's iteration, Hölder regularity and Harnack inequality. In particular, we consider the following second order partial differential equation of Kolmogorov type

$$\sum_{i,j=1}^{m_0} \partial_{x_i} \left(a_{ij}(x,t) \partial_{x_j} u(x,t) \right) + \sum_{i,j=1}^{N} b_{ij} x_j \partial_{x_i} u(x,t) - \partial_t u(x,t) + \\ + \sum_{i=1}^{m_0} b_i(x,t) \partial_i u(x,t) - \sum_{i=1}^{m_0} \partial_{x_i} \left(a_i(x,t) u(x,t) \right) + c(x,t) u(x,t) = 0$$

where $(x,t) = (x_1, \ldots, x_N, t) = z$ is a point of $\mathbb{R}^{N+1}$, and $1 \le m_0 \le N$. (a_{ij}) is an uniformly positive symmetric matrix with bounded measurable coefficients, (b_{ij}) is a constant matrix. We apply the Moser's iteration method to prove the local boundedness of the solution u under minimal integrability assumption on the coefficients.