



UNIVERSITÀ DI PARMA

DIPARTIMENTO DI SCIENZE MATEMATICHE, FISICHE E INFORMATICHE

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Notizie

SEMINARIO

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Filtrations induced by tilting modules

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Abstract

Let A be a ring and $A\text{-Mod}$ the category of left A -modules. The notion of tilting module has been axiomatised in 1979 by Brenner and Butler, generalising that of progenerator for modules of projective dimension 1. Such a classical 1-tilting module T determines a torsion pair $(KE_0(T), KE_1(T))$ on the category of modules, where

$$KE_i(T) = \{M \in A\text{-Mod} : Ext^j(T, M) = 0 \forall j \neq i\}, \quad i = 0, 1.$$

Therefore, each module M admits a filtration with two factors belonging to $KE_i(T)$ for $i = 0, 1$. Miyashita in 1986 extended the tilting notion for modules of projective dimension $n \geq 1$. A classical n -tilting module T naturally gives rise to $n + 1$ classes of modules in $A\text{-Mod}$, called Miyashita classes:

$$KE_i(T) = \{M \in A\text{-Mod} : Ext^j(T, M) = 0 \forall j \neq i\}, \quad i = 0, 1, \dots, n.$$

For $n > 1$, the Miyashita classes are too small in order to filter every left A -module. Working on the derived category $D(A)$, the Miyashita classes can be equivalently described as

$$KE_i(T) = \{M \in A\text{-Mod} : \text{Hom}_{D(A)}(T, M[j]) = 0 \forall j \neq i\}, \quad i = 0, 1, \dots, n.$$

Consider the larger classes

$$\mathcal{KE}_i(T) = \{M^\bullet \in D(A) : \text{Hom}_{D(A)}(T, M^\bullet[j]) = 0 \forall j \neq i\}, \quad i = 0, 1, \dots, n.$$

We provide, for a not necessarily finitely generated n -tilting module T , a decomposition of any module in terms of objects in these classes. This decomposition generalises the one found for $n = 1$: the Miyashita classes can indeed be regarded as the piece of these new classes visible in the category of modules. For $i = 1, \dots, n$, the class $\mathcal{KE}_i(T)$ is the $(-i)$ -shift of the heart of the t -structure \mathcal{T} associated to the tilting module T . The decomposition of any modules is obtained using n torsion pairs in the hearts of as many t -structures linking the natural t -structure to the t -structure \mathcal{T} in $D(A)$.