

# Workshop on Riemannian and Complex Geometry

On the occasion of the seventieth birthday of  
Francesco Mercuri

Parma, September 19-20, 2016

## PROGRAM

MONDAY	TIME	SPEAKER
	14:30 – 15:20	THORBERGSSON
	15:20 – 16:10	JAVALOYES
	16:10 – 16:40	COFFE BREAK
	16:40 – 17:30	NELLI
	17:30 – 18:20	BETTIOL

TUESDAY	TIME	SPEAKER
	10:00 – 10:50	MUSSO
	10:50 – 11:40	GENTILI
	11:40 – 12:10	COFFE BREAK
	12:10 – 13:00	LUQUESIO
	14:30 – 15:20	MARI

## ABSTRACT

**Prof. Graziano Gentili:** *The Mittag-Leffler Theorem for regular functions of a quaternionic variable.*

In this talk we present a version of the classical Mittag-Leffler Theorem for regular functions over quaternions. Our result relies upon an appropriate notion of principal part, that is inspired by the recent definitions of spherical power series expansion and spherical Laurent series expansion for quaternionic regular functions. To place this result in its appropriate setting we preliminary present two different notions of quaternionic analyticity that inspire possible definitions of Laurent series expansions and possible notions of semiregular (meromorphic) functions.

**Dott. Renato Ghini Bettiol:** *Positive curvature and the Bochner technique.*

It is an old question of Hopf whether the product of 2-spheres,  $S^2 \times S^2$ , admits a Riemannian metric with positive sectional curvature. In this talk, I will explain how some nontrivial information about such metrics, if they exist, can be obtained combining the Bochner technique with a trick of Thorpe about modified curvature operators in dimension 4. I will then discuss a curvature positivity condition in higher dimensions inspired by this method. This is joint work with R. Mendes (WWU Münster)

**Prof. P. Jorge Luquecio:** *The Gauss map of complete minimal surface with finite total curvature*

Robert Osserman proved that the image of the Gauss map of a complete, non flat minimal surface in  $\mathbb{R}^3$  with finite total curvature miss at most 3 points. In this paper we prove that the Gauss map of such a minimal immersions omit at most 2 points. This is a sharp result since the Gauss map of the catenoid omits exactly two points. In fact we prove this result for a wider class of iso-metric immersions, that share the basic differential topological properties of the complete minimal surfaces of finite total curvature. This is a join work with Francesco Mercuri.

**Prof. Luciano Mari:** *On the spectrum of minimal submanifolds in space forms*

Let  $\varphi : M^m \rightarrow N^n$  be an immersed minimal submanifold in an ambient space close, in a suitable sense, to the space form  $\mathbb{N}_k^n$  of sectional curvature  $-k \leq 0$ . In this talk, I survey on some recent results obtained in collaboration with various colleagues from Brazil, to ensure that the Laplace-Beltrami operator of  $M$  has purely discrete (respectively, purely essential) spectrum. In the last case, we also give an explicit description of the spectrum. Our criteria apply to many examples of minimal submanifolds constructed in the last 30 years, and answer a question posed by S.T.Yau in his Millenium lectures. The geometric conditions involve the Hausdorff dimension of the limit set of  $\varphi$  and the behaviour at infinity of the density function

$$\Theta(r) = \frac{\text{vol}(M \cap B_r^n)}{\text{vol}(\mathbb{B}_r^m)},$$

where  $B_r^n, \mathbb{B}_r^m$  are geodesic balls of radius  $r$  in  $N^n$  and  $\mathbb{N}_k^m$ , respectively.

**Prof. Emilio Musso:** *Topologically embedded Minding Surface.*

In this talk we will investigate the geometry of topologically embedded Minding surfaces (pseudo-spherical twisted columns). We will show that a pseudo-spherical twisted column is uniquely characterized, up to translations along the screw-axis, by four phenomenological invariants : the helicity,  $\eta \in \mathbb{Z}_2$ , the parity  $\epsilon \in \mathbb{Z}_2$ , the wave number  $n \in \mathbb{N}$  and the aspect ratio  $\mathfrak{d} > 0$ . Furthermore, we exhibit a numerical/symbolical algorithm to reconstruct a pseudo-spherical twisted columns from its invariants.

**Dott. Miguel Àngel Javaloyas:** *Wind Finslerian structures: from Zermelo's navigation to the causality of spacetimes*

The main goal of this talk is to show the relation between two different problems:

- (a) classical Zermelo's navigation problem even when the trajectories of the moving objects (planes, ships) are influenced by strong winds or streams but the wind is stationary (it does not depend on time);
- (b) a natural description of the causal structure of relativistic spacetimes endowed with a non-vanishing Killing vector field (SSTK splittings).

We will show that both problems are equivalent, being possible to solve the first problem with the help of (a generalization) of Finsler Geometry. We will study the causal properties of the spacetimes in Finslerian terms and vice versa. Among the applications, we obtain the solution of Zermelo's navigation with arbitrary stationary wind, metric-type properties (distance-type arrival function, completeness, existence of minimizing, maximizing or closed geodesics), as well as description of spacetime elements (Cauchy developments, black hole horizons) in terms of Finslerian elements in Killing initial data. A general Fermat's principle of independent interest for arbitrary spacetimes, as well as its applications to SSTK spacetimes and Zermelo's navigation, will be also described.

**Prof. Barbara Nelli:** *Old and new results about constant mean curvature surfaces*

We survey some results, by the speaker and not, about constant mean curvature surfaces in different ambient spaces. We will pay special attention at the constant mean curvature PDE's from a geometrical viewpoint.

**Prof. Gudlaugur Thorbergsson:** *Classical symmetric spaces.*

We will give a characterization of the classical compact symmetric spaces as certain Grassmannians of maximally isotropic subspaces.