About a mathematical question in combustion theory and related topics

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The talk will be divided in two parts. In the first one, we will briefly introduce several topics of general interest, such as combustion and flame, reaction-diffusion models, free boundary problems, traveling waves.

In the second part, we will provide a mathematical response to an issue that bothered specialists of combustion theory. We consider a system of two nonlinear reaction-diffusion equations with fractional reaction order α, $0 \leqslant \alpha \leqslant 1$. There are two free boundaries, respectively called the *ignition* and *trailing* interfaces - or there is only one, the ignition boundary. The issue is the following: is there a cut-off value of α at which the trailing interface disappears? The value $\alpha = 1/2$ was predicted.

We look for a special solution that travels at constant velocity $c > 0$. The trick is to reduce the problem to the study of a non-Lipschitz vector field in dimension 2

$$\begin{cases} x'(t) = y(t), \\ \Lambda y'(t) = cy(t) + x^{\alpha}(t). \end{cases} \tag{1}$$

If the time to reach the origin is finite, then the trailing interface exists. We shall see that the actual cut-off is $\alpha = 1$ (see [1,2]).

[1] C.-M. Brauner, R. Roussarie, P. Shang, L. Zhang, *Existence of a traveling wave solution in a free interface problem with fractional order kinetics*, J. Differ. Equ. 281 (2021), 105-147.
[2] R. Roussarie, *Some Applications of the Poincaré-Bendixson Theorem*, Qual. Theory Dyn. Syst. 20 (2021), 1-17.