



Instability and Bifurcation Course Program

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The course is divided into 8 lectures of approximately two hours each, that will be given on Tuesdays and Thursdays from 3pm to 5pm (time in Italy, UTC+1), starting on Thursday Nov. 4th, until Tuesday Nov. 30th. We will give below a short description of each lecture.

1. Nov 4: Introduction, and generalities on geometric variational problems on Riemannian manifold. Riemannian Geometry, affine connections, curvature, geodesics, Jacobi fields and conjugate points. The Morse Index Theorem.
2. Nov 9: An introduction to Bifurcation Theory. Bifurcation from a simple eigenvalue, and the Crandall-Rabinowitz theorem. Variational bifurcation and Morse index. An overview of equivariant bifurcation and symmetry breaking: the Smoller-Waserman theorem.
3. Nov 11: Global bifurcation results. Rabinowitz theorem. The abstract theory will be presented with the discussion of a simple concrete problem, that will be studied in detail: global bifurcation for a class on nonlinear *Yamabe-type* ODEs ([arXiv:2107.08181](https://arxiv.org/abs/2107.08181)).
4. Nov. 16: Bifurcation in the geodesic variational problem: the fixed endpoints case, the free endpoints case, and the periodic case. Timelike and lightlike geodesics in Lorentzian manifolds, with general relativistic interpretation.
5. Nov. 18: Strongly indefinite variational problems in geometry: spectral flow and bifurcation. Geodesics in semi-Riemannian manifolds, Maslov index, nondegenerate conjugate points and bifurcation ([arXiv:math/0211091](https://arxiv.org/abs/math/0211091)).
6. Nov. 23: The Yamabe problem in compact manifolds without boundary. First and second variation of the Hilbert-Einstein functional in conformal classes. Local rigidity. Bifurcation of solutions of the Yamabe problem in product manifolds and in homogeneous submersions ([arXiv:1012.1497](https://arxiv.org/abs/1012.1497)).
7. Nov. 25: The noncompact case: singular Yamabe problem on spheres ([arXiv:1401.7071](https://arxiv.org/abs/1401.7071)) and multiplicity of solutions via bifurcation theory. More existence and multiplicity results in noncompact manifolds ([arXiv:1603.07788](https://arxiv.org/abs/1603.07788)).
8. Nov. 30: An overview of geometric variational problems and bifurcation results: minimal and constant mean curvature surfaces ([arXiv:1306.6043](https://arxiv.org/abs/1306.6043)), metrics with constant Q-curvature ([arXiv:1806.01373](https://arxiv.org/abs/1806.01373)), and, time permitting, a discussion of a problem concerning minimal 2-spheres in ellipsoids.