On weakly Kähler hyperbolic manifolds and geometric applications

Kähler hyperbolic manifolds were introduced by Gromov around thirty years ago. In his seminal paper he showed that this class of Kähler manifolds enjoys several remarkable properties: for instance they are of general type, Kobayashi hyperbolic and their $L^2$-Hodge $(p,q)$-numbers are positive if and only if $p+q$ equals the complex dimension of the manifold. In this talk I will report about a recent joint work with S. Diverio, P. Eyssidieux and S. Trapani where we introduced a weak notion of Kähler hyperbolicity. I will explain the reasons behind our definition and I will show how some of the properties proved by Gromov for Kähler hyperbolic manifolds remain true in our more general setting. As a main application, I will describe how these ideas allow to give a positive answer to the Lang conjecture for Kähler hyperbolic manifolds.