Kähler geometry of holomorphic submersions

Proper holomorphic submersions of Kähler manifolds can be thought of as both a generalisation of holomorphic vector bundles and as a way of studying the behaviour of Kähler manifolds in families. On holomorphic vector bundles, the Hitchin Kobayashi correspondence establishes an equivalence between the existence of special connections, called Hermite-Einstein connection, and an algebro-geometric notion of stability. We will describe a generalisation of the Hermite-Einstein connection on more general fibrations, called optimal symplectic connections, which allow to construct a Kähler metric with constant scalar curvature on the total space. We will then describe a moduli space of fibrations admitting an optimal symplectic connection.