Building BGK operator using ϕ-divergence

Vincent PAVAN *

The Boltzmann equation is an essential model which enables to catch gas flows in many situations when rarefaction occurs and for which usual moment equations are not able to give a fair representation of the phenomena at stakes. However, even taking into account the increasing capacities of parallel computers, the complexity of its numerical resolution makes it impossible to solve in the most complex 3D geometries. Hence the necessity to simplify it by using proper approximations.

In that spirit, two main ideas have been developped. The first one consists in giving moment pictures of the kinetic equation by using a number of moments which is beyond the usual ones (mass, moment, energy and their associated fluxes). This extension is known as the method of Rational Extended Thermodynamics (I.Müller T.Ruggeri [1]) and it has been popularized, in the mathematical community, by C.D. Levermore [3] or H. Struchstrup [2]. The second one consists in keeping the kinetic equation but simplifying the collision integral operator, following in that way the seminal work initiated by Bhatnagar Groos and Krook. The challenge of the first method is to get well posed hyerbolic equations, while the challenge of the BGK approximation consists in keeping the maximum of the essential physical and mathematical properties known for the Boltzmann operator.

A common point of these methods is the minimization of a convex functional entropy under a few linear constraints. However, following the significant work by M. Junk [6], it has been observed that such a problem of minimization is not always well posed, bringing then the risk of leading to the ill-posedness of the approximating equations.

Fortunately, the case of the single gas is now well understood thanks to the famous ES-BGK model and the latter features all the desired properties one can imagine, either to get hyperbolic moment equation or to work in the kinetic framework. However the generalization of this model to more complex situations, like mixture with internal degree of freddom, does not go by itself and in spite of intensive research it seems that the question is still open.

In this seminar we present a new systematic way to build BGK operator using the context of ϕ-divergence framework to enforce entropy minimization. As the main problem of building moment and BGK is the ill-posedness of the minimization problem using the usual entropy $f\ln(f)-f$ the key idea is to replaced this function by an approximation ϕ_N of it (this is exaclt the ϕ-divergence concept which was first introduced by Csizar and used recently in kinetic theory by [4]) so that the new resulting minimization problem is well posed and so the moment eqution or the BGK construction that follow have the maximum of desirable properties.

Though our final goal is to enlarge the construction of the ES-BGK to non isothermal mixture (the case of isothermal mixture has been proposed for instance in [5]) we present the method of ϕ-divergence in the case of the single gas because of its subtle technical aspects.

*Institut Universitaire des Systèmes Thermiques Industriels
5 rue Enrico Fermi
13453 Marseille cedex 13

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